Assignment 6

No need to hand in any problem.

1. Identify the boundary points, interior points, interior and closure of the following sets in $\mathbb{R}$:

   (a) $[1,2)\cup(2,5)\cup\{10\}$.

   (b) $[0,1]\cap\mathbb{Q}$.

   (c) $\bigcup_{k=1}^{\infty}(1/(k+1),1/k)$.

   (d) $\{1,2,3,\cdots\}$.

2. Identify the boundary points, interior points, interior and closure of the following sets in $\mathbb{R}^2$:

   (a) $R\equiv[0,1)\times[2,3)\cup\{0\}\times(3,5)$.

   (b) $\{(x,y): 1<x^2+y^2\le 9\}$.

   (c) $\mathbb{R}^2\setminus\{(1,0),(1/2,0),(1/3,0),(1/4,0),\cdots\}$.

3. Describe the closure and interior of the following sets in $C[0,1]$:

   (a) $\{f:\ f(x)>-1,\ \forall x\in[0,1]\}$.

   (b) $\{f:\ f(0)=f(1)\}$.

4. Let A and B be subsets of (X,d). Show that
$$\overline{A\cup B}=\overline{A}\cup\overline{B}\ .$$
   Is it true that
$$\overline{A\cap B}=\overline{A}\cap\overline{B}\ ?$$

5. Show that $\overline{E}=\{x\in X:\ d(x,E)=0\}$ for every non-empty $E\subset X$.

6. Let $E\subset(X,d)$. Show that E° is the largest open set contained in E in the sense that E° is open and $G\subset E^\circ$ whenever $G\subset E$ is open.

7. Let E be a subset of (X,d). Prove the relation $E^\circ=X\setminus\left(\overline{X\setminus E}\right)$.